

Probabilistic Methods in Combinatorics

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Hints for assignment 11

Problem 1. Note that $\alpha(G)$ is 1-Lipschitz with respect to vertex exposure.

Problem 2*. Let u be the least integer such that $\mathbb{P}(\chi(G) \leq u) \geq \frac{1}{300}$. Let Y be the size a smallest set of vertices S such that $\chi(G \setminus S) \leq u$. First show that there exists some absolute constant λ such that $\mathbb{E}[Y] \leq \lambda\sqrt{n}$. Then show that there exists a constant $c > 0$ such that

$$\mathbb{P}\left[\text{there is a set } S \text{ of size at most } 2\lambda\sqrt{n} \text{ with } \chi(G[S]) \geq \frac{c\sqrt{n}}{\log n}\right] \leq \frac{1}{300}.$$