

# Probabilistic Methods in Combinatorics

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## Hints for assignment 11

**Problem 1.** Note that  $\alpha(G)$  is 1-Lipschitz with respect to vertex exposure.

**Problem 2\*.** Let  $u$  be the least integer such that  $\mathbb{P}(\chi(G) \leq u) \geq \frac{1}{300}$ . Let  $Y$  be the size a smallest set of vertices  $S$  such that  $\chi(G \setminus S) \leq u$ . First show that there exists some absolute constant  $\lambda$  such that  $\mathbb{E}[Y] \leq \lambda\sqrt{n}$ . Then show that there exists a constant  $c > 0$  such that

$$\mathbb{P} \left[ \text{there is a set } S \text{ of size at most } 2\lambda\sqrt{n} \text{ with } \chi(G[S]) \geq \frac{c\sqrt{n}}{\log n} \right] \leq \frac{1}{300}.$$